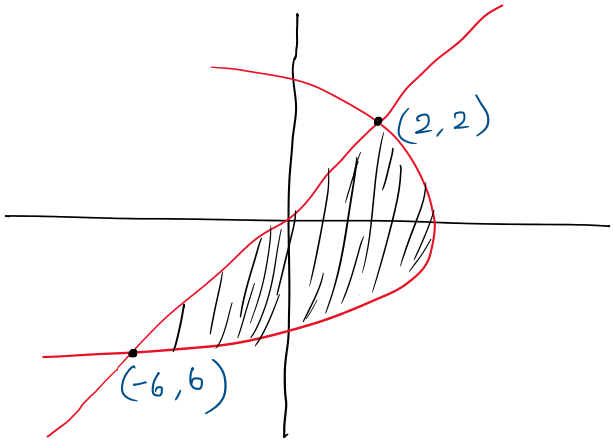
Question :- Find the area enclosed by the line $x = y$ and the parabola $4x + y^2 = 12$.

Solution :- line $x = y$ (given) ——— (1)

and the parabola, $4x + y^2 = 12$ ——— (2)

putting $x = y$ in equation (2).



$$4x + y^2 = 12$$

$$\Rightarrow 4 \cdot y + y^2 = 12$$

$$\Rightarrow y^2 + 4y - 12 = 0$$

$$\Rightarrow y^2 + 6y - 2y - 12 = 0$$

$$\Rightarrow y(y+6) - 2(y+6) = 0$$

$$\Rightarrow (y-2)(y+6) = 0$$

Therefore, $y = 2, -6$.

now, from the equation, $4x + y^2 = 12$

putting $y = 2$, in the above equation,

$$4x + (2)^2 = 12$$

$$\Rightarrow 4x + 4 = 12$$

$$\Rightarrow 4x = 8$$

$$\Rightarrow x = \frac{8}{4}$$

$$\Rightarrow \boxed{x = 2}$$

Now, putting $y = -6$ in $4x + y^2 = 12$

$$\Rightarrow \quad 4x + (-6)^2 = 12$$

$$\Rightarrow \quad 4x + 36 = 12$$

$$\Rightarrow \quad 4x = 12 - 36$$

$$\Rightarrow \quad x = \frac{-24}{4}$$

$$\boxed{x = -6}$$

The obtained co-ordinates are, when, $y = 2, -6$

$(2, 2)$ and $(-6, -6)$

from, $\quad 4x + y^2 = 12$

$$\Rightarrow \quad 4x = 12 - y^2$$

$$\Rightarrow \quad \frac{4x}{4} = \frac{12}{4} - \frac{y^2}{4} \quad \text{(Dividing both side by 4)}$$

$$\Rightarrow \quad x = \left(3 - \frac{y^2}{4}\right)$$

The Area of the curves and the region obtained by the intersection of point $(2, 2)$ and $(-6, -6)$ is given by the integral of the upper curve minus the integral of lower curve over each region.

which is,

$$\text{Area} = \int_{-6}^{2} \left(3 - \frac{y^2}{4}\right) dy - \int_{-6}^{2} y \, dy$$

$$= \int_{-6}^{2} \left(3 - \frac{y^2}{4} - y\right) dy$$

$$\left[\qquad \right]_{-6}^{2}$$

$$= \left[3y - \frac{y^3}{12} - \frac{y^2}{2} \right]_{-6}^{2} \quad \text{(by Integrating)}$$

$$= \left(3 \cdot 2 - \frac{(2)^3}{12} - \frac{(2)^2}{2} \right) - \left(3 \cdot (-6) - \frac{(-6)^3}{12} - \frac{(-6)^2}{2} \right)$$

$$= \left(6 - \frac{8}{12} - \frac{4}{2} \right) - \left(-18 + \frac{216}{12} - \frac{36}{2} \right)$$

$$= \left(6 - \frac{2}{3} - 2 \right) - \left(-18 + 18 - 18 \right)$$

$$= \left(\frac{18 - 2 - 6}{3} \right) + 18$$

$$= \frac{10}{3} + 18 = \frac{10 + 18 \cdot (3)}{3}$$

$$= \frac{10 + 54}{3}$$

$$= \frac{64}{3} \text{ Sq. units}$$

Area $= \frac{64}{3}$ sq. units